## 1 Lecture 4: Summary statistics

## Data Visualization • 1-DAV-105

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### 1.1 Introduction

- Summary statistics (popisné charakteristiky / štatistiky) are quantities that summarize basic properties of a single variable (a table column), such as the mean.
- We can also characterize dependencies between pairs of variables.
- Together with simple plots, such as histograms, they give us the first glimpse at the data when working with a new data set.
- We start by loading the movie data set, which we use to illustrate these terms.


### 1.2 Importing the movie data set

- The same data set as in group tasks 04 .
- The data set describes 2049 movies.
- The data set was downloaded from https://www.kaggle.com/rounakbanik/the-movies-dataset and preprocessed, keeping only movies with at least 500 viewer votes.
[1]:

```
import numpy as np
import pandas as pd
from IPython.display import Markdown
import matplotlib.pyplot as plt
import seaborn as sns
pd.options.display.float_format = '{:,.2f}'.format
```

[2]:

```
url = 'https://bbrejova.github.io/viz/data/Movies_small.csv'
movies = pd.read_csv(url)
display(movies.head())
```

| title | year | budget | revenue | original_language | runtime |
| ---: | ---: | ---: | ---: | ---: | ---: |
| Toy Story | 1995 | $30,000,000.00$ | $373,554,033.00$ | en | 81.00 |
| Jumanji | 1995 | $65,000,000.00$ | $262,797,249.00$ | en | 104.00 |
| Heat | 1995 | $60,000,000.00$ | $187,436,818.00$ | en | 170.00 |
| GoldenEye | 1995 | $58,000,000.00$ | $352,194,034.00$ | en | 130.00 |
| Casino | 1995 | $52,000,000.00$ | $116,112,375.00$ | en | 178.00 |


|  | release_date | vote_average | vote_count |
| :--- | ---: | ---: | ---: |
| 0 | $1995-10-30$ | 7.70 | $5,415.00$ |
| 1 | $1995-12-15$ | 6.90 | $2,413.00$ |
| 2 | $1995-12-15$ | 7.70 | $1,886.00$ |
| 3 | $1995-11-16$ | 6.60 | $1,194.00$ |
| 4 | $1995-11-22$ | 7.80 | $1,343.00$ |

overview
0 Led by Woody, Andy's toys live happily in his ...

```
1 When siblings Judy and Peter discover an encha...
2 Obsessive master thief, Neil McCauley leads a ...
3 James Bond must unmask the mysterious head of ...
4 The life of the gambling paradise - Las Vegas ...
```


### 1.3 Measures of central tendency (miery stredu / polohy)

These represent a typical value in a sample $x$ with values $x_{1}, \ldots, x_{n}$ (one numerical column of a table).

- Mean (priemer) $\bar{x}=\frac{\sum_{i=1}^{n} x_{i}}{n}$
- This is the arithmetic mean, there are also geometric and harmonic means.
- Median (medián) is the middle value when the values ordered from smallest to largest.
- For even $n$ usually defined as the average of the two middle values.
- Median of $10,12,15,16,16$ is 15 .
- Median of $10,12,15,16,16,20$ is 15.5 .
- Mode (modus) is the most frequent value (for a discrete variable).
- Mode of $10,12,15,16,16$ is 16.
- For continuous variables, we may look for a mode in a histogram (this is sensitive to bin size).
https://commons.wikimedia.org/wiki/File:Visualisation_mode_median_mean.svg Cmglee, CC BY-SA 3.0


### 1.3.1 Properties of the measures

- If we apply linear transformation $a \cdot x_{i}+b$ with the same constants $a$ and $b$ to all values $x_{i}$, mean, median and mode will be also transformed in the same way.
- This corresponds e.g. to the change in the units of measurement (grams vs kilograms, degrees C vs degrees F)
- Mean can be heavily influenced by outliers.
- Mean of 800, 1000, 1100, 1200, 1800, 2000 and 30000 is 5414.3, median 1200.
- Mean of $800,1000,1100,1200,1800,2000$ and 10000 is 2557.1 , median 1200.
- Therefore we often prefer median (e.g. median salary).


### 1.3.2 Computation in Pandas

Below we apply functions mean, median, mode to a Series (column year of our table).
Note that mode returns a Series of results (for case of ties). Here just a single value 2013.
Note the use of Python f-strings to print the results.
[3]:

```
display(Markdown("**Properties of the column `year` in our table:**"))
print(f"Mean: {movies['year'].mean():.2f}")
print(f"Median: {movies['year'].median()}")
print(f"Mode:\n{movies['year'].mode()}")
```

Properties of the column year in our table:

Mean: 2004.14
Median: 2008.0
Mode:
02013
Name: year, dtype: int64
Let us see these values in a histogram of the column values (overall view and detail).
[4]:

```
# set up figure with two plots
figure, axes = plt.subplots(1, 2, figsize=(8,3), sharey=True)
# plot histograms, use discrete=True to have each year in one bin
sns.histplot(data=movies, x='year', discrete=True, ax=axes[0])
sns.histplot(x=movies.query('year>=2000') ['year'], discrete=True, ax=axes[1])
# titles and axis labels
axes[0].set_ylabel("The number of movies")
axes[0].set_title('All years in the dataset')
axes[1].set_title('Years 2000-2017')
# compute three summary statics, set up their color and label
stats = [{'label':'mean', 'value':movies['year'].mean(), 'color':'red'},
    {'label':'median', 'value':movies['year'].median(), 'color':'green'},
    {'label':'mode', 'value':movies['year'].mode(), 'color':'black'}]
# add dots for all statistics to both plots (at y=5)
for a in axes:
    for s in stats:
        a.plot(s['value'], 5, 'o', color=s['color'], label=s['label'])
    a.legend()
pass
```



- Functions mean and median can be applied to all numerical columns in a table.
- With axis=1 we can compute means or medians in rows.

```
display(Markdown("**`movies.mean(numeric_only=True)`:**"), movies.
    *mean(numeric_only=True))
display(Markdown("**`movies.median(numeric_only=True)`:**"), movies.
    <median(numeric_only=True))
```

movies.mean(numeric_only=True):
year $\quad 2,004.14$
budget 55,108,939.70
revenue 198,565,134.28
runtime 112.66
vote_average 6.63
vote_count 1,704.64
dtype: float64
movies.median(numeric_only=True):
year $\quad 2,008.00$
budget 38,000,000.00
revenue $\quad 122,200,000.00$
runtime 109.00
vote_average 6.60
vote_count 1,092.00
dtype: float64

### 1.4 Quantiles, percentiles and quartiles (kvantily, percentily, kvartily)

- Median is the middle value in a sorted order.
- Therefore about $50 \%$ of values are smaller and $50 \%$ larger.
- For a different percentage $p$, the $p$-th percentile is at position roughly $(p / 100) \cdot n$ in the sorted order of values.
- Similarly quantile (in Pandas), but we give fraction between 0 and 1 rather than percentage.
- Specifically quartiles are three values $Q_{1}, Q_{2}$ and $Q_{3}$ that split input data into quarters.
- Therefore, $Q_{2}$ is the median.
- Many definitions exist regarding situations when the desired fraction falls between two values (we can take lower, higher, mean, weighted mean etc).


### 1.4.1 Computation in Pandas

- Function quantile gets a single value between 0 and 1 or a list of values and returns corresponding quantiles.
- To get quantiles for $0.1,0.2, \ldots, 0.9$, we generate a regular sequence of values using np. arange.

```
display(Markdown("**Median:**"), movies['year'].median())
display(Markdown("**Quantile for 0.5:**"), movies['year'].quantile(0.5))
display(Markdown("**All quartiles:**"), movies['year'].quantile([0.25, 0.5, 0.
    475]))
display(Markdown("**With step 0.1:**"), movies['year'].quantile(np.arange(0.1,\sqcup
    41, 0.1)))
```


## Median:

2008.0

## Quantile for 0.5 :

2008.0

## All quartiles:

$0.252,000.00$
0.50 2,008.00
0.75 2,013.00

Name: year, dtype: float64
With step 0.1:
0.10 1,988.80
0.20 1,998.00
0.30 2,002.00
0.40 2,005.00
0.50 2,008.00
0.60 2,010.00
$0.702,012.00$
$0.802,014.00$
0.90 2,015.00

Name: year, dtype: float64
The code below plots the quartiles highlighted in a histogram.
[7]:

```
# setup histograms
figure, axes = plt.subplots(1, 2, figsize=(8,3), sharey=True)
sns.histplot(data=movies, x='year', discrete=True, ax=axes[0])
sns.histplot(x=movies.query('year>=1985') ['year'], discrete=True, ax=axes[1])
axes[0].set_ylabel("The number of movies")
axes[0].set_title('All years in the dataset')
axes[1].set_title('Years 1985-2017')
# compute and display quartiles
quartiles = movies['year'].quantile([0.25, 0.5, 0.75])
for a in axes:
    a.plot(quartiles, [5] * len(quartiles), 'o', color='black')
pass
```



- The code below illustrates how the quantile function works when returning quantiles which do not correspond to a single input value.
- Optional parameter interpolation accepts values 'linear' (default), 'lower', 'higher', 'midpoint', 'nearest'.
- Imagine the lowest element at quantile 0 , the highest element at quantile 1 and the rest evenly spaced between. The quantile at position between two elements is influenced only by its two neighbors.

For example, consider list of values $[0,10,20,100]$. * Values taken from the list: $p=0: 0, p=1 / 3$ : 10, $p=2 / 3: 20, p=1: 100 *$ Example of a different value: $p=1 / 4$ : by default linear interpolation at $3 / 4$ between 0 and 10, i.e. 7.5.

Note that linear interpolation is continuous as $p$ changes from 0 to 1 .
[8] :

```
a = pd.Series([0, 100])
b = pd.Series([0, 10, 20, 30, 100])
c = pd.Series([0, 10, 20, 100])
quantiles = [0.01, 0.25, 0.5, 0.75]
display(Markdown(f"**Quantiles for {list(a)}**"), a.quantile(quantiles))
display(Markdown(f"**Quantiles for {list(b)}**"), b.quantile(quantiles))
display(Markdown(f"**Quantiles for {list(c)}**"), c.quantile(quantiles))
display(Markdown(f"**Quantiles for {list(c)} with `interpolation='lower'`**"),
c.quantile(quantiles, interpolation='lower'))
```


## Quantiles for $[0,100]$

$0.01 \quad 1.00$
$0.25 \quad 25.00$
$0.50 \quad 50.00$
$0.75 \quad 75.00$
dtype: float64
Quantiles for $[0,10,20,30,100]$
$0.01 \quad 0.40$
$0.25 \quad 10.00$
$0.50 \quad 20.00$

```
0.75 30.00
dtype: float64
```

Quantiles for $[0,10,20,100]$

| 0.01 | 0.30 |
| :--- | :---: |
| 0.25 | 7.50 |
| 0.50 | 15.00 |
| 0.75 | 40.00 |
| dtype: | float 64 |

Quantiles for $[0,10,20,100]$ with interpolation='lower'

| 0.01 | 0 |
| :--- | :---: |
| 0.25 | 0 |
| 0.50 | 10 |
| 0.75 | 20 |
| dtype: | int 64 |

### 1.5 Measures of variability (miery variability)

- Values in the sample may be close to their mean or median, or they can spread widely.
- It is important to consider how representative is the mean or median of the whole set.

Examples of measures:

- Range of values from minimum to maximum (sensitive to outliers).
- Interquartile range IQR (kvartilové rozpätie): range between $Q_{1}$ and $Q_{3}$ (contains the middle half of the data).
- Variance and standard deviation (described next).


### 1.5.1 Variance and standard deviation (rozptyl a smerodajná odchýlka)

## Variance

- For each value in the sample compute its difference from the mean and square it: $\left(x_{i}-\bar{x}\right)^{2}$.
- After squaring, we get non-negative values (and squares are easier to work with mathematically than absolute values).
- Variance is the mean of these squares, but we divide by $n-1$ rather than $n$ :

$$
s^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n-1}
$$

- We divide by $n-1$ rather than $n$, because we would otherwise underestimate the true variance of the underlying population (more in the statistics course).
- For large $n$, the difference between dividing by $n$ and $n-1$ is negligible.


## Standard deviation

- Square root of the variance

$$
s=\sqrt{s^{2}}
$$

- It is expressed in the same units as the original values (variance is in units squared).


## Properties

- Larger variance and standard deviation mean that data are spread farther from the mean
- If we apply linear transformation $a \cdot x_{i}+b$ with the same constants $a$ and $b$ to all values $x_{i}$ :
- Neither variance nor standard deviation change with $b$.
- Variance is multiplied by $a^{2}$, standard deviation by $|a|$.
- These measures are strongly influenced by outliers:
- For $800,1000,1100,1200,1800,2000,30000$ st. dev. is 10850.0, IQR 850.
- For $800,1000,1100,1200,1800,2000,10000$ st. dev. is 3310.5, IQR 850.


### 1.5.2 Computation in Pandas

We can use functions min, max, std, var, which work similarly to mean.
[9] :

```
display(Markdown("**Minimum**"), movies['year'].min())
display(Markdown("**Maximum**"), movies['year'].max())
display(Markdown("**Mean**"), movies['year'].mean())
display(Markdown("**Variance**"), movies['year'].var())
display(Markdown("**Standard deviation**"), movies['year'].std())
q1 = movies['year'].quantile(0.25)
q3 = movies['year'].quantile(0.75)
display(Markdown("**Q1, Q3 and interquartile range:**"), q1, q3, q3-q1)
```


## Minimum

1927
Maximum
2017

## Mean

2004.1449487554905

## Variance

161.2714600681735

## Standard deviation

12.699270060447313

## Q1, Q3 and interquartile range:

2000.0
2013.0
13.0

### 1.6 Outliers (odl'ahlé hodnoty)

- Outliers are the values which are far from the typical range of values.
- In data analysis, it is important to check the outliers.
- If they represent errors, we may try to correct or remove them.
- They can also represent interesting anomalies.
- Different definitions of outliers may be appropriate in different situations.
- The criterion by statistician John Tukey is often used:
- Outliers are the values outside of the range $Q_{1}-k \cdot I Q R, Q_{3}+k \cdot I Q R$, e.g. for $k=1.5$.
- In our example $800,1000,1100,1200,1800,2000,30000$ :
$-Q_{1}=1050, Q_{3}=1900, I Q R=850$.
$-Q_{1}-1.5 \cdot I Q R=-225, Q_{3}+1.5 \cdot I Q R=3175$.
- Outliers are values smaller than -225 or larger than 3175 ; here only 30000 .
- The range of outliers is not influenced if we change outliers values (as long as they stay outside of range Q1-Q3).


### 1.6.1 Computation in Pandas

- The code below finds outliers in the year column.
- We compute the lower and upper thresholds manually from quartiles.
- Then we use query to select rows and count how many there are.
- Function count counts the values in a Series or columns of a DataFrame, ignoring missing values.
[10]:

```
# get quartiles and iqr
q1 = movies['year'].quantile(0.25)
q3 = movies['year'].quantile(0.75)
iqr = q3 - q1
# compute thresholds for outliers
lower = q1 - 1.5 * iqr
upper = q3 + 1.5 * iqr
# count outliers
count = movies.query('year < @lower or year > @upper')['year'].count()
# print results
display(Markdown(f"**Outliers outside of range:** [{lower}, {upper}]"))
display(Markdown(f"**Outlier count:** {count}"))
display(Markdown(f"**Total count:** {movies['year'].count()}"))
```

Outliers outside of range: [1980.5, 2032.5]
Outlier count: 112
Total count: 2049

### 1.7 Boxplot (krabicový graf)

- Boxplots were developed by Mary Eleanor Hunt Spear and John Tukey.
- For a single numerical variable it shows the five-number summary consisting of the minimum, $Q_{1}$, median $\left(Q_{2}\right), Q_{3}$ and the maximum.
- Median is shown as a thick line, $Q_{1}$ and $Q_{3}$ as a box and minimum and maximum as "whiskers".
- Outliers are often excluded from the whiskers and shown as individual points.
- Summaries of different samples are often compared in a single boxplot.
- Boxplots allow clear comparison of basic characteristics.


### 1.7.1 Boxplots in Seaborn

- We use boxplot finction from Seaborn.
- Below is a simple horizontal boxplot of the year column.
- Recall that quartiles are 2000, 2008 and 2013, minimum 1927, maximum 2017, outliers outside of [1980.5, 2032.5].
[11]:

```
axes = sns.boxplot(data=movies, x='year')
axes.figure.set_size_inches(8,2)
```



- Below is a vertical boxplot of the year column split into groups according to language.
- This is achieved by specifying both x and y options.
[12]:

```
sns.boxplot(data=movies, x='original_language', y='year')
pass
```



- Below we draw a strip plot on top of the boxplot.
- This allows us to see both individual data points and the summary.
- Here it does not work very well for en, better suited for smaller datasets.
- We see that some languages have extremely low number of points, boxplots not ideal in that case.
[13]:

```
axes = sns.boxplot(data=movies, x='original_language', y='year', color='C1')
sns.stripplot(data=movies, x='original_language', y='year', color='C0',
    alpha=0.5, size=5, jitter=0.2)
axes.figure.set_size_inches(10,6)
pass
```



### 1.8 Quick overview of a data set: describe in Pandas

Function describe gives a quick overview of a data set with many statistics described today.
[14](%5Cbegin%7Btabular%7D%7Blrrrrr%7D):

```
movies.describe()
```

\& year \& budget \& revenue \& runtime \& vote_average <br>
count \& $2,049.00$ \& $1,959.00$ \& $1,965.00$ \& $2,049.00$ \& $2,049.00$ <br>
mean \& $2,004.14$ \& $55,108,939.70$ \& $198,565,134.28$ \& 112.66 \& 6.63 <br>
std \& 12.70 \& $53,139,663.86$ \& $233,028,732.94$ \& 24.76 \& 0.77 <br>
min \& $1,927.00$ \& 1.00 \& 15.00 \& 7.00 \& 4.00 <br>
$25 \%$ \& $2,000.00$ \& $16,000,000.00$ \& $52,882,018.00$ \& 97.00 \& 6.10 <br>
$50 \%$ \& $2,008.00$ \& $38,000,000.00$ \& $122,200,000.00$ \& 109.00 \& 6.60 <br>
$75 \%$ \& $2,013.00$ \& $75,000,000.00$ \& $250,200,000.00$ \& 124.00 \& 7.20 <br>
$\max$ \& $2,017.00$ \& $380,000,000.00$ \& $2,787,965,087.00$ \& 705.00 \& 9.10
\end{tabular}

|  | vote_count |
| :--- | ---: |
| count | $2,049.00$ |
| mean | $1,704.64$ |
| std | $1,607.89$ |
| min | 501.00 |
| $25 \%$ | 709.00 |
| $50 \%$ | $1,092.00$ |

$$
\begin{array}{rr}
75 \% & 2,000.00 \\
\max & 14,075.00
\end{array}
$$

- By default describe only considers numerical columns.
- Other columns can be included by include='all'.
- Different statistics reported for categorical columns (unique, top, freq).
[15](%5Cbegin%7Btabular%7D%7Blrr%7D):

```
movies.describe(include='all').transpose()
```

\& count \& unique <br>
title \& 2049 \& 2018 <br>
year \& $2,049.00$ \& NaN <br>
budget \& $1,959.00$ \& NaN <br>
revenue \& $1,965.00$ \& NaN <br>
original_language \& 2049 \& 16 <br>
runtime \& $2,049.00$ \& NaN <br>
release_date \& 2049 \& 1740 <br>
vote_average \& $2,049.00$ \& NaN <br>
vote_count \& $2,049.00$ \& NaN <br>
overview \& 2049 \& 2049
\end{tabular}

|  |  | top | freq |
| :---: | :---: | :---: | :---: |
| title |  | Beauty and the Beast | 3 |
| year |  | NaN | NaN |
| budget |  | NaN | NaN |
| revenue |  | NaN | NaN |
| original_language |  | en | 1958 |
| runtime |  | NaN | NaN |
| release_date |  | 2014-12-25 | 6 |
| vote_average |  | NaN | NaN |
| vote_count |  | NaN | NaN |
| overview | Led by Woody, Andy's toys | e happily in his ... | 1 |


|  | mean | std | min | $25 \%$ |
| :--- | ---: | ---: | ---: | ---: |
| title | NaN | NaN | NaN | NaN |
| year | $2,004.14$ | 12.70 | $1,927.00$ | $2,000.00$ |
| budget | $55,108,939.70$ | $53,139,663.86$ | 1.00 | $16,000,000.00$ |
| revenue | $198,565,134.28$ | $233,028,732.94$ | 15.00 | $52,882,018.00$ |
| original_language | NaN | NaN | NaN | NaN |
| runtime | 112.66 | 24.76 | 7.00 | 97.00 |
| release_date | NaN | NaN | NaN | NaN |
| vote_average | 6.63 | 0.77 | 4.00 | 6.10 |
| vote_count | $1,704.64$ | $1,607.89$ | 501.00 | 709.00 |
| overview | NaN | NaN | NaN | NaN |
|  |  |  |  |  |
|  | $50 \%$ | $75 \%$ |  | max |
| title | NaN | NaN |  | NaN |


| year | $2,008.00$ | $2,013.00$ | $2,017.00$ |
| :--- | ---: | ---: | ---: |
| budget | $38,000,000.00$ | $75,000,000.00$ | $380,000,000.00$ |
| revenue | $122,200,000.00$ | $250,200,000.00$ | $2,787,965,087.00$ |
| original_language | NaN | NaN | NaN |
| runtime | 109.00 | 124.00 | 705.00 |
| release_date | NaN | NaN | NaN |
| vote_average | 6.60 | 7.20 | 9.10 |
| vote_count | $1,092.00$ | $2,000.00$ | $14,075.00$ |
| overview | NaN | NaN | NaN |

### 1.9 Correlation (korelácia)

- We are often interested in relationships among different variables (data columns).
- We will see two correlation coefficients that measure the strength of such relationships.
- Beware: correlation does not imply causation.
- If electricity consumption grows in a very cold weather, there might be cause-and-effect relationship: the cold weather is causing people to use more electricity for heating.
- If healthier people tend to be happier, which is the cause and which is effect?
- Both studied variables can be also influenced by some third, unknown factor. For example, within a year, deaths by drowning increase with increased ice cream consumption. Both increases are spurred by warm weather.
- The observed correlation can be just a coincidence, see the Redskins rule and a specialized webpage Spurious Correlations.
- You can easily find such "coincidences" by comparing many pairs of variables (a practice called data dredging).


### 1.9.1 Pearson correlation coefficient

- It measures linear relationship between two variables.
- Consider pairs of values $\left(x_{1}, y_{1}\right), \ldots,\left(x_{n}, y_{n}\right)$, where $\left(x_{i}, y_{i}\right)$ are two different features of the same object.

$$
r=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}} \sqrt{\sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)^{2}}}
$$

- Or equivalently:

$$
r=\frac{1}{n-1} \sum_{i=1}^{n}\left(\frac{x_{i}-\bar{x}}{s_{x}}\right)\left(\frac{y_{i}-\bar{y}}{s_{y}}\right) .
$$

- where $s_{x}$ is the standard deviation of variable $x$.
- Expression $\left(x_{i}-\bar{x}\right) / s_{x}$ is called the standard score or z-score, and it tells us how many standard deviations above or below the mean value $x_{i}$ is.
- The product of $\left(x_{i}-\bar{x}\right) / s_{x}$ and $\left(y_{i}-\bar{y}\right) / s_{y}$ is positive if $x_{i}$ and $y_{i}$ lie on the same side of the respective means of $x$ and $y$ and negative if they lie on the opposite sides.


### 1.9.2 Properties of Pearson correlation coefficient

Values of Pearson correlation coefficient

- The value of $r$ is always from interval $[-1,1]$.
- It is 1 if $y$ grows linearly with $x,-1$ if $y$ decreases linearly with increasing $x$.
- Zero means no correlation.
- Values between 0 and 1 mean intermediate value of positive correlation, values between -1 and 0 negative correlation.
https://commons.wikimedia.org/wiki/File:Correlation_coefficient.png Kiatdd, CC BY-SA 3.0


## Some cautions

- Pearson correlation measures only linear relationships ( x and y in the bottom row have nonlinear relationships but their correlation is 0 ).
- Pearson correlation does not depend on the slope of the best-fit line (see the middle row below).
https://commons.wikimedia.org/wiki/File:Correlation_examples2.svg public domain


## Other properties

- Pearson correlation does not change if we linearly scale each variable, i.e. $a x_{i}+b, c y_{i}+d$ (for $a, c>0)$.
- Pearson correlation is symmetric $r_{x, y}=r_{y, x}$.


### 1.9.3 Linear regression

- The process of finding the line best representing the relationship of $x$ and $y$ is called linear regression.
- It can be used in higher dimensions to predict one variable as a linear combination of many others.
- You will study linear regression in later courses, but we may draw regression lines in some plots.
[16]:

```
x = [10, 8, 13, 9, 11, 14, 6, 4, 12, 7, 5]
y1 = [8.04, 6.95, 7.58, 8.81, 8.33, 9.96, 7.24, 4.26, 10.84, 4.82, 5.68]
sns.regplot(x=x, y=y1)
pass
```



### 1.9.4 Spearman's rank correlation coefficient

- It can detect non-linear relationships.
- We first convert each variable into ranks:
- Rank of $x_{i}$ is its index in the sorted order of $x_{1}, \ldots, x_{n}$.
- Equal values get the same (average) rank.
- For example, the ranks of $10,0,10,20,10,20$ are $3,1,3,5.5,3,5.5$.
- Then we compute Pearson correlation coefficient of the two rank sequences.
- Values of $1,-1$ if $y$ monotonically increases or decreases with $x$.
- It is less sensitive to distant outliers (actual values of $x$ and $y$ are not important).
https://commons.wikimedia.org/wiki/File:Spearman_fig1.svg Skbkekas, CC BY-SA 3.0


### 1.9.5 Computation in Pandas

Function corr computes correlation between all pairs of numerical columns. There is also a version to compare two Series.

In our table, the highest Pearson correlation is 0.69 for pairs (budget, revenue), (vote_count, revenue)
[17](%5Cbegin%7Btabular%7D%7Blrrrrrr%7D):

```
movies.corr(numeric_only=True)
```

\& year \& budget \& revenue \& runtime \& vote_average \& vote_count <br>
year \& 1.00 \& 0.28 \& 0.12 \& -0.07 \& -0.34 \& 0.12 <br>
budget \& 0.28 \& 1.00 \& 0.69 \& 0.22 \& -0.18 \& 0.47 <br>
revenue \& 0.12 \& 0.69 \& 1.00 \& 0.25 \& 0.06 \& 0.69 <br>
runtime \& -0.07 \& 0.22 \& 0.25 \& 1.00 \& 0.31 \& 0.25 <br>
vote_average \& -0.34 \& -0.18 \& 0.06 \& 0.31 \& 1.00 \& 0.33 <br>
vote_count \& 0.12 \& 0.47 \& 0.69 \& 0.25 \& 0.33 \& 1.00
\end{tabular}

With Spearman rank correlation, the correlation between revenue and budget remains similar, but correlation between vote_count and budget decreases from 0.69 to 0.56 .
[18]: movies.corr(method='spearman', numeric_only=True)
[18]:

|  | year | budget | revenue | runtime | vote_average | vote_count |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| year | 1.00 | 0.21 | 0.02 | -0.03 | -0.27 | 0.14 |
| budget | 0.21 | 1.00 | 0.68 | 0.24 | -0.28 | 0.37 |
| revenue | 0.02 | 0.68 | 1.00 | 0.21 | -0.08 | 0.56 |
| runtime | -0.03 | 0.24 | 0.21 | 1.00 | 0.32 | 0.27 |
| vote_average | -0.27 | -0.28 | -0.08 | 0.32 | 1.00 | 0.29 |
| vote_count | 0.14 | 0.37 | 0.56 | 0.27 | 0.29 | 1.00 |

- Here we illustrate the regression line for revenue versus vote_count.
- We use Seaborn regplot to draw scatterplot together with the regression line.
- Points are made smaller and transparent by scatter_kws=\{'alpha':0.7, 's':5\}.
- The plot on the right shows ranks instead of actual values.
- Ranks are computed using rank function for Series.
- Pearson correlation coefficient is probably driven by outliers.
[19]:

```
# figure with two plots
figure, axes = plt.subplots(1, 2, figsize=(10,5))
# plot of values
sns.regplot(x=movies['revenue'] / 1e6, y=movies['vote_count'],
    ax=axes[0], scatter_kws={'alpha':0.7, 's':5})
axes[0].set_xlabel('revenue in millions')
axes[0].set_ylabel('vote count')
# compute ranks
revenue_rank = movies['revenue'].rank()
vote_count_rank = movies['vote_count'].rank()
# plot of ranks
sns.regplot(x=revenue_rank, y=vote_count_rank,
    ax=axes[1], scatter_kws={'alpha':0.7, 's':5})
axes[1].set_xlabel('rank of revenue')
axes[1].set_ylabel('rank of vote count')
pass
```




### 1.10 Anscombe's quartet and importance of visualization

- Anscombe's quartet are four artificial data sets designed by Francis Anscombe.
- All have the same or very similar values of means and variances of both $x$ and $y$, Pearson correlation coefficient (0.816) and linear regression line.
- But visually we see each has a very different character.
- The bottom row illustrates the influence of outliers on correlation and regression.
- Overall this shows that plots give us a much better idea of the properties of a data set than simple numerical summaries.
[20]:

```
# adapted from https://matplotlib.org/stable/gallery/specialty_plots/anscombe.
    chtml
x = [10, 8, 13, 9, 11, 14, 6, 4, 12, 7, 5]
y1 = [8.04, 6.95, 7.58, 8.81, 8.33, 9.96, 7.24, 4.26, 10.84, 4.82, 5.68]
y2 = [9.14, 8.14, 8.74, 8.77, 9.26, 8.10, 6.13, 3.10, 9.13, 7.26, 4.74]
y3 = [7.46, 6.77, 12.74, 7.11, 7.81, 8.84, 6.08, 5.39, 8.15, 6.42, 5.73]
x4 = [8, 8, 8, 8, 8, 8, 8, 19, 8, 8, 8]
y4 = [6.58, 5.76, 7.71, 8.84, 8.47, 7.04, 5.25, 12.50, 5.56, 7.91, 6.89]
datasets = [(x, y1), (x, y2), (x, y3), (x4, y4)]
figure, axes = plt.subplots(2, 2, sharex=True, sharey=True, figsize=(7, 7))
axes[0, 0].set(xlim=(0, 20), ylim=(2, 14))
for ax, (x, y) in zip(axes.flat, datasets):
    ax.plot(x, y, 'o')
    # linear regression
    slope, intercept = np.polyfit(x, y, deg=1)
    ax.axline(xy1=(0, intercept), slope=slope, color='gray')
```



### 1.10.1 Visual overview of a data set: pairplot in Seaborn

- Seaborn pairplot generates a matrix of plots for all numerical columns.
- The diagonal contains histograms of individual columns.
- Off-diagonal entries are scatterplots of two columns.
- Here only 3 columns shown for simpler examination.
[21]:

```
subset = movies.loc[:, ['vote_count', 'budget', 'revenue']]
grid = sns.pairplot(subset, height=2.5)
pass
```



### 1.11 Computing summaries of subsets of data: groupby from Pandas

- We have seen that Seaborn can create plots where data are split into groups according to a categorical variable.
- One example are boxplots, which we have seen today.
- How can we compute summary statistics for each such group in Pandas?
[22]:

```
sns.boxplot(data=movies, x='original_language', y='year')
pass
```



- Pandas DataFrame supports function groupby which splits the table into groups based on values of some column.
- We can apply a summary statistics function on each group.
- Below we compute medians of all numerical columns for each language and show the first 5 languages.
[23](%5Cbegin%7Btabular%7D%7Blrrrr%7D):

```
movies.groupby('original_language').median(numeric_only=True).head()
```

\& \multicolumn{2}{c}{ year } \& budget \& revenue <br>
original_language \& \& \& \& <br>
cn \& $2,006.00$ \& $12,902,809.00$ \& $39,388,380.00$ \& 108.50 <br>
da \& $2,010.00$ \& $10,000,000.00$ \& $16,740,418.00$ \& 119.00 <br>
de \& $2,003.50$ \& $6,250,000.00$ \& $70,000,000.00$ \& 129.00 <br>
en \& $2,008.00$ \& $40,000,000.00$ \& $126,397,819.00$ \& 109.00 <br>
es \& $2,007.00$ \& $2,000,000.00$ \& $30,448,000.00$ \& 118.00 <br>
\& \& \& \& <br>
vote_average \& vote_count \& <br>
original_language \& \& \& \& <br>
cn \& 7.20 \& 762.50 \& <br>
da \& 6.80 \& 867.50 \& <br>
de \& 7.60 \& 669.00 \& <br>
en \& 6.60 \& $1,126.00$ \& <br>
es \& 7.60 \& 797.00 \&
\end{tabular}

- We can also apply describe on the groupby groups.
- Here only two numerical columns of the original table are shown.
[24](!%5B%5D(./images/4601cea72bd58b149e5fa35c1a511418_245_769_246_1863.jpg)):

```
subset = movies.loc[:, ['original_language', 'year', 'budget']]
subset.groupby('original_language').describe().head()
```

original_language

| cn | $2,010.00$ | 3.00 | $14,872,795.67$ | $4,479,793.25$ | $11,715,578.00$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| da | $2,013.00$ | 5.00 | $13,440,000.00$ | $12,369,640.25$ | $3,800,000.00$ |
| de | $2,013.00$ | 8.00 | $18,223,718.75$ | $30,623,544.47$ | $1,530,000.00$ |
| en | $2,017.00$ | $1,891.00$ | $56,637,200.97$ | $53,394,829.52$ | 1.00 |
| es | $2,014.00$ | 5.00 | $7,500,000.00$ | $8,046,738.47$ | $1,500,000.00$ |


|  | $25 \%$ |  | $50 \%$ | $75 \%$ | max |
| :--- | ---: | ---: | ---: | ---: | ---: |
| original_language |  |  |  |  |  |
| cn | $12,309,193.50$ | $12,902,809.00$ | $16,451,404.50$ | $20,000,000.00$ |  |
| da | $7,400,000.00$ | $10,000,000.00$ | $11,000,000.00$ | $35,000,000.00$ |  |
| de | $4,100,000.00$ | $6,250,000.00$ | $15,084,937.50$ | $92,620,000.00$ |  |
| en | $18,000,000.00$ | $40,000,000.00$ | $80,000,000.00$ | $380,000,000.00$ |  |
| es | $2,000,000.00$ | $2,000,000.00$ | $13,000,000.00$ | $19,000,000.00$ |  |

### 1.12 Summary

We have seen several summary statistics:

- mean, median, mode
- percentiles, quantiles, quartiles
- min, max, interquartile range, variance, standard deviation
- Pearson and Spearman correlation

Visualization:

- boxplot
- scatter plots with regression lines
- pairplot

Pandas:

- functions for computing statistics, describe
- groupby
- next week: more Pandas

More details in a statistics course.

